

## ANALYSIS OF WAVEGUIDE POST CONFIGURATIONS: PART 3

## INFLUENCE OF GENERAL WAVEGUIDE TERMINATIONS

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Analysis of general post configurations in an infinitely long rectangular waveguide was presented earlier<sup>1,2</sup>. This technical note considers the extension of that work to the case when the two waveguide ports are generally terminated.

Introduction

Two general multiple gap post structures in infinite rectangular waveguide were analysed recently<sup>1</sup> using the reaction concept. The structures considered were  $N$  posts with one gap in each and a single post with  $N$  gaps. General expressions for the gap impedance matrix elements were derived for both cases.

In some practical cases, however, the waveguide arms will not both be match-terminated. It is thus of interest to study the influence of general terminations on the impedance matrix elements for these general post configurations.

General N-Post Structure

Fig. 1 is the schematic representation of a general  $N$ -post configuration with the two waveguide arms terminated in reflection coefficients  $\rho_{amn}$  and  $\rho_{bmn}$ . The post geometry parameters are as defined in Fig. 1 of Ref. 1.

This configuration can be represented in the form of an  $N \times N$  impedance matrix containing self and mutual impedance terms for the various ports.

For the general post  $i$  the self-impedance expression for the  $n$ th spatial harmonic mode  $Z_{Tin}$  is equivalent to the reaction between the unit current excitation in post  $i$  and the electric field produced by the current at the post location<sup>1,3</sup>. This electric field expression for the terminated waveguide case differs from the infinite waveguide case by a factor  $\tau_{iimn}$ . This factor can be arrived at from first principles by considering multiple reflections from the two waveguide arms. Thus, for  $i = 1$  to  $N$

$$\tau_{iimn} = \frac{\left[1 + \rho_{amn} \exp(-2\Gamma_{mn} L_{ai})\right] \left[1 + \rho_{bmn} \exp(-2\Gamma_{mn} L_{bi})\right]}{1 - \rho_{amn} \rho_{bmn} \exp\{-2\Gamma_{mn} (L_{ai} + L_{bi})\}} \quad (1)$$

This factor is identical to that given by Eisenhart and Khan<sup>4</sup> for a single post terminated guide case. Although the approach differs slightly from<sup>4</sup> the usefulness of this technique will be apparent later.

The factor  $\tau_{iimn}$  is carried in all the intervening mathematical steps<sup>1</sup> resulting in the self-impedance expression of the gap associated with the general post  $i$ . Thus

$$Z_{Tin} = \sum_{m=1}^{\infty} Z_{mn} \tau_{iimn} \left\{ \frac{K_{ipm}}{K_{ign}} \right\}^2 \quad (2)$$

The mutual impedance term for two general posts  $i$  and  $j$  for a given  $n$ ,  $Z_{Tijn}$ , is equivalent to the reaction between the unit current excitation in post  $i$  and the electric field produced at post  $i$  due to a unit current excitation in post  $j$ <sup>1,3</sup>. Again, it can be shown from first principles that the electric field produced by the post  $j$  current at post  $i$  location differs from the infinite waveguide value by a factor  $\tau_{ijmn}$  given by, for  $i, j = 1$  to  $N$

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$$\tau_{ijmn} = \frac{\left[1 + \rho_{amn} \exp(-2\Gamma_{mn} L_{ai})\right] \left[1 + \rho_{bmn} \exp(-2\Gamma_{mn} L_{bi})\right]}{1 - \rho_{amn} \rho_{bmn} \exp\{-2\Gamma_{mn} (L_{ai} + L_{bi} + L_{ij})\}} \quad (3)$$

Again, this factor is carried in all the intervening steps resulting in the mutual impedance expression  $Z_{Tijn}$  as

$$Z_{Tijn} = \sum_{m=1}^{\infty} Z_{mn} \tau_{ijmn} \left\{ \frac{K_{ipm} K_{jpm}}{K_{ign} K_{jgn}} \right\} \exp\{-\Gamma_{mn} L_{ij}\} \quad (4)$$

An examination of Eqs. 1 to 4 and Fig. 1 indicates that Eq. 4 reduces to Eq. 2 when  $i = j$ . Thus Eq. 4 is the general expression for the impedance matrix elements for each  $n$  for a terminated guide.

The reciprocal nature of the network manifests itself in the equivalence

$$Z_{Tijn} = Z_{Tjin}, \quad i \neq j \quad (5)$$

General Single Post N Gap Structure

It can be shown following an identical approach that the general expression for the gap admittance matrix for each  $n$  is given by

$$Y_{Tin} = \left[ \sum_{m=1}^{\infty} Z_{mn} \tau_{iimn} \frac{K_{ipm}^2}{K_{ign} K_{gn}} \right]^{-1} \quad (6)$$

where  $\tau_{iimn}$  reduces to the form shown in Eq. 1.

For single post and coplanar multi-post structures all  $L_{ij} = 0$  and the expressions assume a simpler form.

Staggered Two-Post Structure

For this structure a comparison can be made between the obstacle networks obtained for the infinite waveguide case and the generally terminated case. The complete network between the two waveguide arms is shown in Fig. 2. This network is similar to that for an infinite waveguide (Fig. 3 of Ref. 2) except that the network elements  $Z(pqr)$  and  $Y(ABC)$  have been replaced by their terminated counterparts  $Z_T(pqr)$  and  $Y_T(ABC)$  respectively<sup>2</sup>. The dominant  $H_{10}$  mode coupling network comprises the elements  $Y(uvw)$  which are the same as the infinite guide case<sup>2</sup>. In addition, each waveguide arm is terminated in an admittance element

$$G_{ki} + jB_{ki} \quad (k = a, i = 1 \text{ & } k = b, i = 2)$$

where

$$G_{ki} = \frac{1 - \rho_{ki0}^2}{1 + \rho_{ki0}^2 + 2\rho_{ki0} \cos 2\beta L_{ki}} Y_{ci0} \quad (7)$$

$$B_{ki} = \frac{2\rho_{ki0} \sin 2\beta L_{ki}}{1 + \rho_{ki0}^2 + 2\rho_{ki0} \cos 2\beta L_{ki}} Y_{ci0} \quad (8)$$

References

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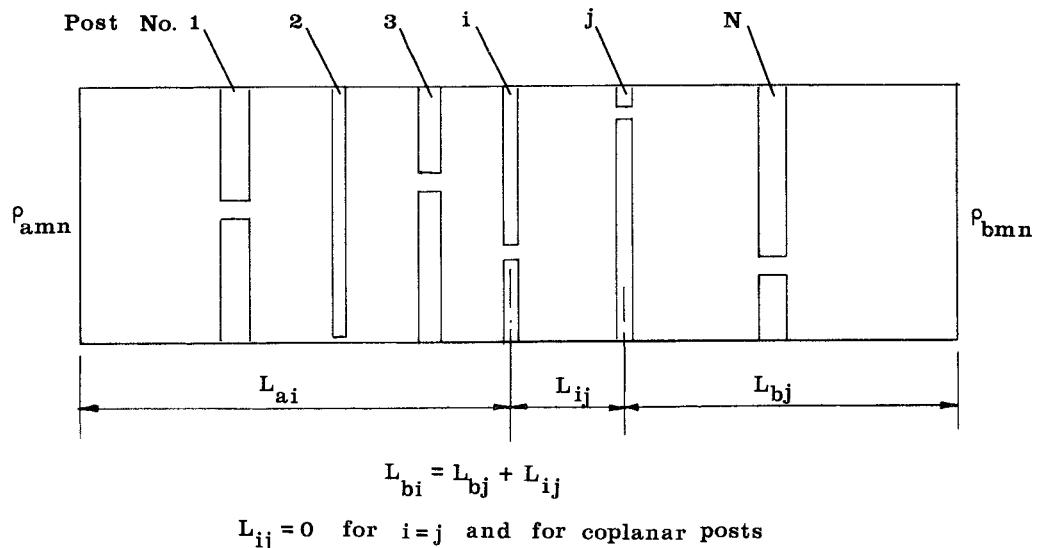


Fig. 1 Schematic diagram of a general  $N$  post structure in a rectangular waveguide. The waveguide arms are terminated in reflection coefficients  $\rho_{amn}$  and  $\rho_{bmN}$ . Two general post  $i$  and  $j$  are shown.  $L_{ai}$  is the effective distance of termination  $a$  from the post plane  $i$  and  $L_{bj}$  is the effective distance of termination  $b$  from the post plane  $j$ .  $L_{ij}$  is the axial separation between posts  $i$  and  $j$ . The terminations must, of course, be outside the extremities of the post complex.

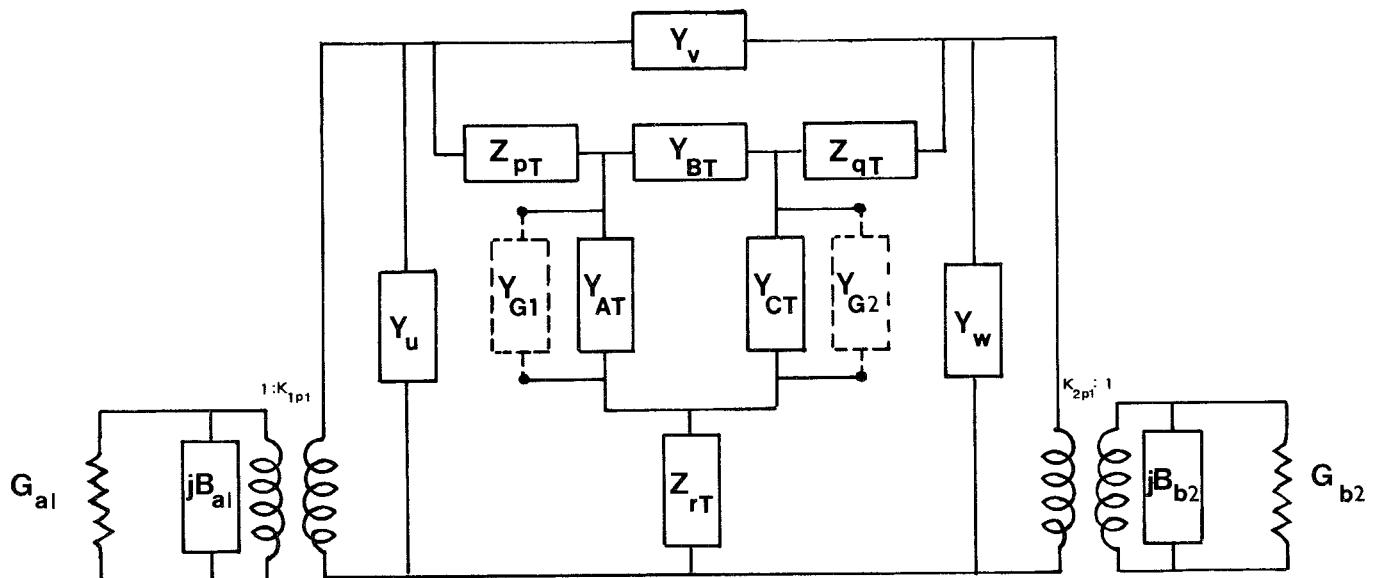


Fig. 2 Network between the two waveguide arms for the dominant  $H_{10}$  mode in the guide.